

Volume Control in a Hybrid Mock Circuit of the Cardiovascular System using Command Filtered Backstepping Approach

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Abstract—Cardiovascular diseases often necessitate implantable therapies such as mechanical circulatory support devices, which are evaluated using mock circulatory systems. This study presents two nonlinear control strategies—classical backstepping and command-filtered backstepping for regulating the hydraulic dynamics of a hybrid pneumo-hydraulic mock circuit. After introducing the design steps of the two controllers, the closed-loop stability analyses are presented in the context of Lyapunov theory. To show the effectiveness of the two controllers, some simulations are performed by also evaluating in terms of tracking error and robustness to measurement noise, and command-filtered backstepping provides the best balance of performance and efficiency for precise, safe device testing.

Keywords—Nonlinear Control, Backstepping, Mock Circulatory Loop.

I. INTRODUCTION

Cardiovascular diseases (CVDs) remain the leading cause of mortality worldwide [1]. While certain conditions can be managed through lifestyle modification, pharmacotherapy, or surgical intervention, advanced pathologies—such as malignant arrhythmias, valvular and arterial lesions, end-stage heart failure, and peripheral arterial disease—frequently necessitate implantable solutions, including vascular prostheses, mechanical circulatory support devices, grafts, and stents [2]. The development of these biomedical devices typically follows a structured progression: virtual design and simulation (in silico), experimental evaluation in physical test platforms (in vitro), preclinical validation in live animals (in vivo), and, ultimately, clinical trials in humans (in clinico). If deficiencies are identified at any stage, the design undergoes revision, and the testing cycle is repeated [3]. Consequently, early identification of potential limitations is essential to streamline development, reduce costs, and minimize resource expenditure prior to market approval.

To enhance physiological fidelity during physical testing, mock circuits are employed to replicate cardiovascular behavior under both normal and pathological conditions. Cardiovascular system hybrid mock circuits (MCs) combine numerical

and physical analogues in a hardware-in-the-loop configuration, enabling the controlled emulation of hemodynamic variables such as flow, pressure, and volume, as well as parameters including resistance and compliance [4]. In this framework, the numerical model generates reference pressure signals (e.g., left ventricular and aortic pressures), which are transmitted to the physical circuit via an electro-hydraulic interface, where they are tracked and regulated. Flow control ensures reservoir volume stability by adjusting pump flow rates, thereby preventing overflow or depletion and maintaining consistent experimental conditions [5].

Various MC architectures have been developed employing hydraulic, mechanical, or pneumatic actuation. Mechanical and hydraulic actuators provide simplicity in modeling and control but are hindered by inertia-related limitations, which result in sluggish responses and reduced accuracy [6] [7]. Pneumatic actuators, in contrast, exhibit faster response times and improved reliability. However, their compressibility introduces nonlinear dynamics that complicate both system modeling and controller design [8].

In order to solve these, a pneumo-hydraulic hybrid MC in which the hydraulic and pneumatic subsystems were controlled using integrator backstepping and Lyapunov-based error linearization algorithms are proposed, respectively, in [9]. This design successfully reproduced a range of physiological and pathological states, including normal, failing, and ventricular assist-device-supported conditions, with high tracking fidelity. On the other hand, the classical backstepping control is insufficient in terms of attenuating measurement noise, so the command-filtered backstepping controller delivers superior accuracy with reduced computational demand in noisy environments.

This study extends the paper in [9] by applying two nonlinear control strategies—classical backstepping and command-filtered backstepping—to regulate the hydraulic compartment of the hybrid MC. The objective of this paper is to compare their tracking accuracy, robustness, and computational efficiency under varying operating conditions.

II. CARDIOVASCULAR SYSTEM MOCK CIRCUIT

The mock circuit (MC) employed in this study integrates a seven-node cardiovascular numerical model that was coupled to a physical pneumo-hydraulic rig via a hardware-in-the-loop interface. Pressure–volume relationships in the left ventricle (LV) and aorta (AO) defined the dynamics, and the model supplied reference LV/AO pressures to regulate the hardware. The setup comprised two pneumo-hydraulic reservoirs (LV and AO) reproducing LVAD preload/afterload, a continuous-flow LVAD pump, and a volume-displacement gear pump between reservoirs for circulation and level balancing. Solenoid valves modulated air inflow/outflow to track the references, while pressure transducers and electromagnetic flow probes measured reservoir pressures and pump flows in real time. Sensor data were acquired and processed in MATLAB/Simulink; further details on dynamics, signal processing, and control appear in [9].

This study focuses on the hydraulic subsystem dynamics and introduces the conceptual design of two backstepping method based controllers implemented to regulate the fluid level difference between the reservoirs. All governing equations and control expressions are derived in MATLAB and implemented in Simulink to simulate system behavior and evaluate controller performance.

III. SYSTEM DYNAMICS

Combining the volume difference dynamics given in [9] and the gear pump (the actuator) dynamics given in [5] the system dynamics can be expressed as

$$\Delta \dot{V}_h = 2Q_{vad} + 2R_{bf}\Delta P - 2\alpha_\omega\omega \quad (1a)$$

$$\dot{\omega} = \frac{1}{J}(-\tau_1 \text{sgn}(\omega) - \alpha_P \Delta P - \tau_2 \omega + K_T I) \quad (1b)$$

$$\dot{I} = \frac{1}{L}(-K_{emf}\omega - R_{gp}I + V_{in}). \quad (1c)$$

where ΔV_h is the fluid level difference, Q_{lvad} is the flow rate generated by the lvad, α_ω is the gear pump stroke volume, R_{bf} is back flow hydraulic resistance, J is the motor inertia, ω is the motor rotation speed, τ_1 is the dry friction constant, τ_2 is the viscous friction constant, K_T is the torque constant, α_P is the load pressure constant, L is the motor inductance, I is the motor current, K_{emf} is the back-electromotive force voltage constant, R_{gp} is the resistance of the motor winding, and V_{in} is the motor input voltage.

For notational simplicity, the system is parameterized as follows: $x_1 = \Delta V$, $x_2 = \omega$, $x_3 = I$, $u = V_{in}$, $f_1 = 2Q_{lvad} - 2R_{bf}\Delta P$, $\theta_1 = \frac{1}{J}$, and $\theta_2 = \frac{1}{L}$, $f_2 = -\tau_1 \text{sgn}(\omega) - \alpha_s \Delta P$, $g_1 = -2\alpha_s \omega$, $g_2 = -\tau_2$, $g_3 = K_T$, $g_4 = -K_{emf}$, and $g_5 = -R_{gp}$.

These assignments yield the simplified dynamical model:

$$\dot{x}_1 = f_1 + g_1 x_2 \quad (2a)$$

$$\dot{x}_2 = \theta_1 f_2 + \theta_1 g_2 x_2 + \theta_1 g_3 x_3 \quad (2b)$$

$$\dot{x}_3 = \theta_2 g_4 x_2 + \theta_2 g_5 x_3 + \theta_2 u. \quad (2c)$$

To design the control system, the tracking error (z_i) definitions are introduced as $z_i = \phi_i - x_i$ where ϕ_1 is the user-defined

reference value for x_1 , usually equal to 0, and ϕ_i (for $i = 2, 3$) is the virtual control signal for the corresponding states. Substituting Equations (2) into the time derivative of tracking error yields the tracking error dynamics:

$$\dot{z}_1 = \dot{\phi}_1 - f_1 - g_1(\phi_2 - z_2) \quad (3a)$$

$$\dot{z}_2 = \dot{\phi}_2 - \theta_1 f_2 - \theta_1 g_2(\phi_2 - z_2) - \theta_1 g_3(\phi_3 - z_3) \quad (3b)$$

$$\dot{z}_3 = \dot{\phi}_3 - \theta_2 g_4(\phi_2 - z_2) - \theta_2 g_5(\phi_3 - z_3) - \theta_2 u. \quad (3c)$$

A. Classical Backstepping

The first control method employs classical backstepping, in which a Lyapunov function is selected to establish stability and ensure convergence of the tracking errors. We begin by choosing a positive definite candidate Lyapunov function as

$$V = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2}z_3^2. \quad (4)$$

Combining Equations (3) and the time derivative of Equation (4) yields

$$\begin{aligned} \dot{V} = & z_1 [\dot{\phi}_1 - f_1 - g_1(\phi_2 - z_2)] \\ & + z_2 [\dot{\phi}_2 - \theta_1 f_2 - \theta_1 g_2(\phi_2 - z_2) - \theta_1 g_3(\phi_3 - z_3)] \\ & + z_3 [\dot{\phi}_3 - \theta_2 g_4(\phi_2 - z_2) - \theta_2 g_5(\phi_3 - z_3) - \theta_2 u]. \end{aligned} \quad (5)$$

The controller signals are designed as follows:

$$\phi_2 = \frac{1}{g_1}(\dot{\phi}_1 - f_1 + k_1 z_1) \quad (6a)$$

$$\phi_3 = \frac{1}{\theta_1 g_3}(\dot{\phi}_2 + g_1 z_1 - \theta_1 f_2 - \theta_1 g_2 \phi_2 + \theta_1 g_2 z_2 + k_2 z_2) \quad (6b)$$

$$\begin{aligned} u = & \frac{1}{\theta_2}(\dot{\phi}_3 + \theta_1 g_3 z_2 - \theta_2 g_4 \phi_2 + \theta_2 g_4 z_2 - \theta_2 g_5 \phi_3 \\ & + \theta_2 g_5 z_3 + k_3 z_3) \end{aligned} \quad (6c)$$

where $k_i > 0$ for $i = 1, 2, 3$ are the control gains and $\dot{\phi}_2$ and $\dot{\phi}_3$ are defined as the derivative of Equations (6a) and (6b) respectively. Substituting Equations (6) into Equation (5) yields $\dot{V} = -k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2$. In this condition since $\dot{V}(0) = 0$ and $\dot{V}(z) < 0 \forall z \neq 0$, $\dot{V}(z)$ is a negative definite function. Therefore, it can be seen that the closed-loop system is globally asymptotically stable in the sense of Lyapunov.

B. Command Filtered Backstepping

This section incorporates a command filtering-based backstepping controller to improve tracking performance in the presence of measurement noise. The filtered tracking error compensates the influence of noise on the system, and a control law is derived to guarantee stability [10]. Because the initial steps of command-filtered backstepping mirror those of classical backstepping, we proceed from Equations (6). Define the filtered tracking error and the compensated tracking error, respectively, as

$$\tilde{z}_i = \phi_{ic} - x_i \quad (7)$$

$$v_i = \tilde{z}_i - \xi_i \quad (8)$$

where ϕ_{ic} is the filtered command signal and ξ_i is the compensated virtual control signal for the corresponding state for $i = 1, 2, 3$. The pseudocontrol signals can be defined as

$$\bar{\phi}_2 = \frac{1}{g_1}(\dot{\phi}_{1c} - f_1 + k_1 \tilde{z}_1) \quad (9a)$$

$$\bar{\phi}_3 = \frac{1}{\theta_1 g_3}(\dot{\phi}_{2c} + g_1 v_1 - \theta_1 f_2 - \theta_1 g_2 \bar{\phi}_2 + \theta_1 g_2 v_2 + k_2 \tilde{z}_2) \quad (9b)$$

$$\bar{u} = \frac{1}{\theta_2}(\dot{\phi}_{3c} + \theta_1 g_3 v_2 - \theta_2 g_4 \bar{\phi}_2 + \theta_2 g_4 v_2 - \theta_2 g_5 \bar{\phi}_3 + \theta_2 g_5 v_3 + k_3 \tilde{z}_3). \quad (9c)$$

Solving Equations (9) for the command derivatives yields

$$\dot{\phi}_{1c} = g_1 \bar{\phi}_2 + f_1 - k_1 \tilde{z}_1 \quad (10a)$$

$$\dot{\phi}_{2c} = \theta_1 g_3 \bar{\phi}_3 - g_1 v_1 + \theta_1 f_2 + \theta_2 g_4 v_2 + \theta_1 g_2 \bar{\phi}_2 - \theta_1 g_2 v_2 - k_2 \tilde{z}_2 \quad (10b)$$

$$\dot{\phi}_{3c} = \theta_2 \bar{u} - \theta_2 g_3 v_2 + \theta_2 g_4 \bar{\phi}_2 - \theta_2 g_4 v_2 + \theta_2 g_5 \bar{\phi}_3 - \theta_2 g_5 v_3 - k_3 \tilde{z}_3. \quad (10c)$$

The command filter is implemented as a low-pass filter in state-space form to attenuate noise:

$$\dot{X} = \begin{bmatrix} 0 & \omega_n \\ -\omega_n & -2\zeta\omega_n \end{bmatrix} X + \begin{bmatrix} \omega_n \\ 0 \end{bmatrix} U \quad (11a)$$

$$Y = \begin{bmatrix} 1 & 0 \\ 0 & \omega_n \end{bmatrix} X \quad (11b)$$

where ω_n is the filter natural frequency, ζ is the filter damping ratio, X is the filter state vector, $U = \bar{\phi}_i$ and $Y = [\phi_{ic} \quad \dot{\phi}_{ic}]^T$. Thus, the filter receives the pseudocontrol signals as input and outputs the filtered command and its derivative. Substituting Equations (10) and (8) into the time derivative of Equation (7) gives the filtered-error dynamics:

$$\dot{\tilde{z}}_1 = -k_1 \tilde{z}_1 + g_1(\bar{\phi}_2 - \phi_{2c} + \xi_2 + v_2) \quad (12a)$$

$$\dot{\tilde{z}}_2 = -k_2 \tilde{z}_2 + \theta_1 g_2(\bar{\phi}_2 - \phi_{2c} + \xi_2) + \theta_1 g_3(\bar{\phi}_3 - \phi_{3c} + \xi_3 + v_3) - g_1 v_1 \quad (12b)$$

$$\dot{\tilde{z}}_3 = -k_3 \tilde{z}_3 + \theta_2 g_4(\bar{\phi}_2 - \phi_{2c} + \xi_2) + \theta_2 g_5(\bar{\phi}_3 - \phi_{3c} + \xi_3) - g_3 v_2 + \theta_2(\bar{u} - u). \quad (12c)$$

Substituting Equations (12) in the time derivative of Equation (8) yields the compensated tracking error dynamics as

$$\dot{v}_1 = -k_1 \tilde{z}_1 + g_1(\bar{\phi}_2 - \phi_{2c} + \xi_2 + v_2) - \dot{\xi}_1 \quad (13a)$$

$$\dot{v}_2 = -k_2 \tilde{z}_2 + \theta_1 g_2(\bar{\phi}_2 - \phi_{2c} + \xi_2) + \theta_1 g_3(\bar{\phi}_3 - \phi_{3c} + \xi_3 + v_3) - g_1 v_1 - \dot{\xi}_2 \quad (13b)$$

$$\dot{v}_3 = -k_3 \tilde{z}_3 + \theta_2 g_4(\bar{\phi}_2 - \phi_{2c} + \xi_2) + \theta_2 g_5(\bar{\phi}_3 - \phi_{3c} + \xi_3) - g_3 v_2 + \theta_2(\bar{u} - u) - \dot{\xi}_3. \quad (13c)$$

Define a candidate Lyapunov function for the compensated tracking errors as

$$V_f = \frac{1}{2} v_1^2 + \frac{1}{2} v_2^2 + \frac{1}{2} v_3^2. \quad (14)$$

In this condition, the candidate Lyapunov function is radially unbounded and positive definite. Combining Equations (13) and the time derivative of Equation (14) yields

$$\begin{aligned} \dot{V}_f = & v_1 \left[-k_1 \tilde{z}_1 + g_1(\bar{\phi}_2 - \phi_{2c} + \xi_2 + v_2) - \dot{\xi}_1 \right] \\ & + v_2 \left[-k_2 \tilde{z}_2 + \theta_1 g_2(\bar{\phi}_2 - \phi_{2c} + \xi_2) \right. \\ & \quad \left. + \theta_1 g_3(\bar{\phi}_3 - \phi_{3c} + \xi_3 + v_3) - g_1 v_1 - \dot{\xi}_2 \right] \\ & + v_3 \left[-k_3 \tilde{z}_3 + \theta_2 g_4(\bar{\phi}_2 - \phi_{2c} + \xi_2) - g_3 v_2 \right. \\ & \quad \left. + \theta_2 g_5(\bar{\phi}_3 - \phi_{3c} + \xi_3) + \theta_2(\bar{u} - u) - \dot{\xi}_3 \right]. \end{aligned} \quad (15)$$

The filtered controller signals are devised as follows:

$$\dot{\xi}_1 = -k_1 \xi_1 + g_1(\bar{\phi}_2 - \phi_{2c} + \xi_2) \quad (16a)$$

$$\dot{\xi}_2 = -k_2 \xi_2 + \theta_1 g_2(\bar{\phi}_2 - \phi_{2c} + \xi_2) + \theta_1 g_3(\bar{\phi}_3 - \phi_{3c}) \quad (16b)$$

$$\dot{\xi}_3 = -k_3 \xi_3 + g_3 v_2 + \theta_2 g_4(\bar{\phi}_2 - \phi_{2c} + \xi_2) + \theta_2 g_5(\bar{\phi}_3 - \phi_{3c} + \xi_3) - g_3 v_2 + \theta_2(\bar{u} - u) \quad (16c)$$

where $k_i > 0$ for $i = 1, 2, 3$ are the control gains. Substituting Equations (16) into Equation (15) yields $\dot{V}_f = -k_1 v_1^2 - k_2 v_2^2 - k_3 v_3^2$. In this condition, since $\dot{V}_f(z)$ is a negative definite function. Therefore, the compensated system is also globally asymptotically stable in the sense of Lyapunov. Finally, substituting Equations (7) and (8) into Equations (9) yields the pseudocontrol derivatives in their implementable form:

$$\dot{\phi}_{1c} = g_1 \bar{\phi}_2 + f_1 - k_1(\phi_{1c} - x_1) \quad (17a)$$

$$\dot{\phi}_{2c} = \theta_1 g_3 \bar{\phi}_3 - g_1(\phi_{1c} - \xi_1 - x_1) + \theta_1 f_2 + \theta_1 g_2 \bar{\phi}_2 - \theta_1 g_2(\phi_{2c} - \xi_2 - x_2) - k_2(\phi_{2c} - x_2) \quad (17b)$$

$$\dot{\phi}_{3c} = \theta_2 \bar{u} - \theta_2 g_3(\phi_{2c} - \xi_2 - x_2) + \theta_2 g_4 \bar{\phi}_2 + \theta_2 g_5 \bar{\phi}_3 - \theta_2 g_4(\phi_{2c} - \xi_2 - x_2) - \theta_2 g_5(\phi_{3c} - \xi_3 - x_3) - k_3(\phi_{3c} - x_3). \quad (17c)$$

Note that ϕ_{ic} is obtained by filtering the pseudocontrol signals and ξ_i is obtained by integrating Equations (16). Consequently, all variables in Equations (17) are computable from the system states and known parameters.

IV. SIMULATION WORKS

The system and controller dynamics were implemented separately in Simulink to test the proposed strategies. First, the nominal (noise-free) plant was simulated. For the Classical Backstepping controller, additive noise was injected into measured signals to emulate real-world disturbances and benchmark performance. The Command-Filtered Backstepping controller was then tested under identical noise conditions, enabling a direct robustness comparison.

All controllers produced input commands within 0–24 V. Figure 1 shows the input signals (top) and actuator responses (bottom). While steady-state behavior was similar, noise in Classical Backstepping caused sharp peaks and dips due to

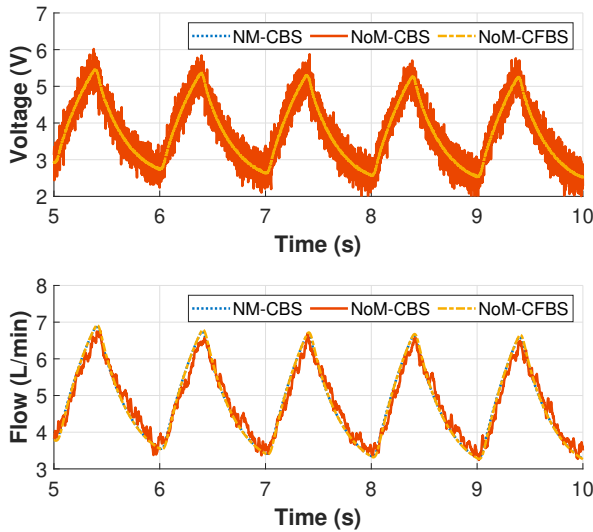


Fig. 1: Control signals over time and the corresponding actuator responses. Here, NM represents the nominal model, NoM the model with signal noise, CBS the classical backstepping, and CFBS the command-filtered backstepping.

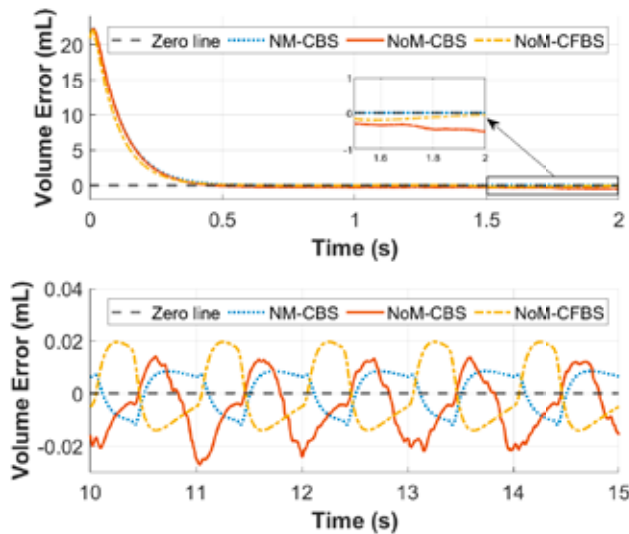


Fig. 2: Transient and Steady-state tracking error.

differentiation of noisy signals, leading to actuator chatter and potential instability. In contrast, Command-Filtered Backstepping smoothed inputs by filtering states before control computation, yielding more stable actuator responses. Figure 2 presents tracking-error signals for both transient (top) and steady-state (bottom) phases.

Across all cases, the inter-reservoir volume difference stayed within limits. However, under noise with Classical Backstepping, oscillations and steady-state error appeared. Command-Filtered Backstepping improved tracking accuracy, with only a slight phase shift. Specifically, Classical Backstepping achieved an average error of 1.1×10^{-3} mL in the nominal case but degraded to -4.4×10^{-3} mL with noise, while Command-Filtered Backstepping reduced the error to 6.2×10^{-4} mL under noise. These results confirm that the command-

filtered approach suppresses noise-induced oscillations and steady-state error.

V. CONCLUSIONS

Classical and command-filtered backstepping controllers were evaluated under identical conditions in the hydraulic subsystem of the hybrid pneumo-hydraulic mock circuit. For both control structures, the closed-loop stability is established. Each approach generated similar control inputs within the 0–24 V range and maintained tracking within acceptable limits. When measurement noises were introduced, the command-filtered backstepping controller achieved performance comparable to that of the classical backstepping controller operating in an ideal, noise-free setting. These findings demonstrate the effectiveness of the command-filtered backstepping architecture for systems subject to sensor noise. In future work, we plan to subject this approach to benchtop tests to assess its real-time performance and robustness.

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