



Synchrosqueezing Transform in Biomedical Applications: A mini review

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Abstract— Time-frequency representation (TFR) provides a good analysis for periodic signals; however, they are insufficient for nonstationary signals. The synchrosqueezing transform (SST) provides a strong analysis of nonstationary signals. The signal has different synchrosqueezing transformations that are implemented using different TFR. This paper provides a review of the different SST methods implemented using different TFR available in the literature, a comparison of these, and their use with different techniques in biomedical signal processing applications. Adding different techniques to the applied SST method affects the signal processing and classification ability of the selected SST method.

Keywords—Biomedical Applications, Synchrosqueezing, Signal Processing, Time-Frequency.

I. INTRODUCTION

The signal representation area has always used frequency notation and Fourier analysis today is the essential tool for analyzing or processing signals and images [1]. The Fourier transform (FT) allows us to model or simulate stationary Gauss random fields easily. While this technique can be expanded to obtain the frequency spectrum of a signal, it requires a full explanation of the signal's behavior over time. Therefore, FT is insufficient for nonstationary signals [2]. Especially for efficient analysis or processing of these signals, local transformations called TFRs must be used. Because these signals are poorly represented by traditional methods that motivate time-frequency (TF) analysis. According to the FT analysis, the TF method has high resolution in time and frequency to facilitate analysis and interpretation. It also allows real-time applications with lower computational complexity to represent the time required to represent and process a signal in a TF plane. It is known that it is a better method in the FT method with these features.

TFRs provide a powerful tool for the analysis of time-series signals. The most popular among traditional TFs is basically a short-time Fourier transform (STFT) that windows the signal around the time of interest before applying FT. Another popular TFR is the continuous wavelet transform (CWT), which shares many features with STFT but is based on a different frequency resolution. Both transformations for these signals are known to draw lines in TF or time-scale (TS) planes [3].

SST, proposed by Daubechies and Maes as a phase-based technique, is a reassignment method [4]. According to traditional TF analysis methods, the SST method can achieve excellent resolution both in frequency and time. The purpose of this method is to increase the readability of the TFR issued

by the CWT using the reassignment procedure and additionally allows the mod to be rolled back. It is important in analyzing the signals in the biomedical field, as it is an effective technique for obtaining improved TFRs while allowing mode recovery of SST.

SST can be extended to a variety of applications. Atmospheric noise suppression [5], seismic data analysis [6] fault diagnosis of a wind turbine [7], and detection for earthquake-damaged structure [8]. In addition to these, it is frequently used in biomedical signal processing applications, and it is developed with different techniques and guides further studies.

This paper demonstrates the use of different techniques in biomedical applications in various fields by comparing different SST methods. First, we define the theories of STFT based synchrosqueezing (FSST), wavelet-based synchrosqueezing (WSST), multivariate synchrosqueezing (MSST), and de-shaped synchrosqueezing (dsSST) methods. After that, we provide a detailed review of which biomedical signal processing applications are used for each of these methods. Finally, a comparison of the SST methods mentioned was made according to the results in the studies.

II. THEORY AND BIOMEDICAL APPLICATIONS

A. Short-Time Fourier Transform based Synchrosqueezing Transform (FSST)

Let's assume that an f signal exists. The behavior of the signal in the frequency domain is examined using the FT of the f signal. In (1), the FT of the signal is represented by $\hat{f}(v)$.

$$\hat{f}(v) = \int_R f(x)e^{-2i\pi vx} dx \quad (1)$$

Analysis with the FT is insufficient for nonstationary signals. For the analysis of such signals, it is necessary to add the time profile and create a TFR. A time profile can be added using windowing. STFT of the signal is obtained by windowing the signal and taking the FT. STFT provides the local frequency information of the signal [9]. The TFR of the f signal obtained by using STFT is shown as V_f . The formulated form is as follows:

$$V_f(\eta, t) = \int_R f(\tau)g(\tau - t)e^{-2i\pi\eta(\tau - t)} d\tau \quad (2)$$

In the synchrosqueezing process, the conversion from the TF profile to the time-local instantaneous frequency profile is made. With this transformation, an ideal TFR is aimed. Local instantaneous frequency information of the signal at time t , frequency η is shown as $\hat{\omega}_f(\eta, t)$ in (3).

$$\hat{\omega}_f(\eta, t) = \frac{1}{2\pi} \partial_t \arg V_f(\eta, t) = \operatorname{Re} \left(\frac{1}{2\pi} \frac{\partial_t V_f(\eta, t)}{V_f(\eta, t)} \right) \quad (3)$$

Formulation of the FSST obtained using the Dirac distribution of TFR and local instantaneous frequency information is as in (4).

$$T_f(\omega, t) = \frac{1}{g(\omega)} \int_R V_f(\eta, t) \delta(\omega - \hat{\omega}_f(\eta, t)) d\eta \quad (4)$$

An example of the FSST of the electroencephalogram (EEG) signal is shown in Fig. 1.

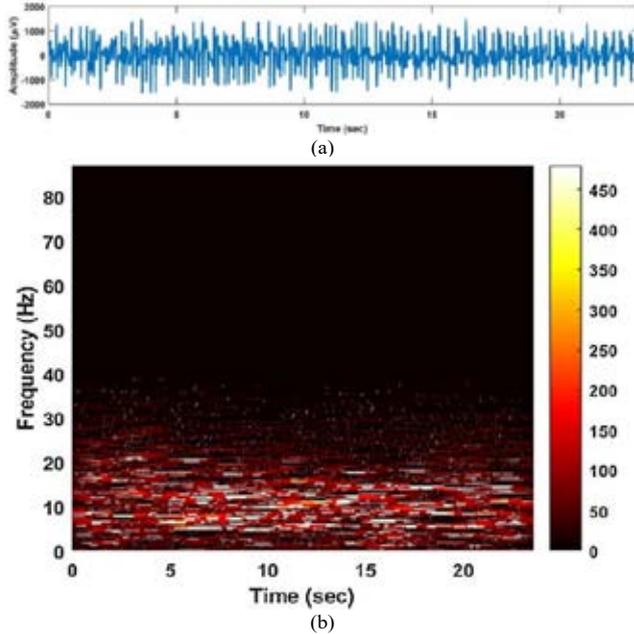


Fig. 1. a)EEG Signal b)FSST representation of the signal [10].

FSST offers an ideal time-local instant frequency representation, despite the presence of unstable noise [11]. There are biomedical applications of the FSST method in the literature. Successful results were achieved by using this transformation in optimizing the predictions of the instant heart rate. In the study conducted in 2016, SST was used to extract the instant heart rate (IHR) and it was concluded that IHR derived from electrocardiogram (ECG) provides an accurate prediction with this method. A comparison of this SST method with the time-dependent auto-regressive model was made and it was observed that the SST method gave more accurate and reliable results. Also, in the study, although it is assumed to be a noise-resistant method, it was concluded that there is not an adequate method in deterministic noises and it should be supported with additional methods [12]. In the literature, there are studies in which this method affects the classification success by using different classification methods. In a study conducted in 2018, it was aimed to classify the epilepsy phase with the information obtained from EEG signals. After applying FSST to EEG signals, a new vector was created with the properties of the matrixes obtained by separating them into 5 segments and taking the grayscale pictures. The classification step was started by selecting the knowledgeable features using the infinite latent feature selection (ILFS) method. Support vector machine (SVM) and k-Nearest Neighbor (kNN) classifier were used in the classification step, a method was

created to determine effective epilepsy phases [10]. FSST method was used as a new approach in the analysis of complex and noisy physiological waveforms such as peripheral venous pressure (PVP) waveform. It was concluded that SST provides more visible and less contaminated properties for the non-fixed hemodynamic PVP waveform. It has also been shown to assist in extracting more physiological dynamic information [13].

B. Wavelet-based Synchrosqueezing Transform (WSST)

CWT is a TFR algorithm that evolves a series of finite energy oscillations called wavelet that shown as $\psi(t)$ [14]. In synchrosqueezing, the CWT of the s signal, defined as a properly selected wavelet ψ , starts at W_s and $W_s(a, b)$ to obtain a concentrated TF picture. It separates again, so instant frequency lines can be removed [15]. In (5), the formula with the corresponding $s(t)$ signal and wavelet coefficients of $W_s(a, b)$ is shown below. Scale factor a located in (5) shifts the wave at frequency ψ so that the oscillation properties at different frequency scales are captured.

$$W_s(a, b) = \int s(t) a^{-\frac{1}{2}} \psi\left(\frac{t-b}{a}\right) dt \quad (5)$$

To motivate the idea, let's assume that it started with a completely harmonic signal; $s(t) = A \cos(\omega t)$. When $\hat{\psi}$, which is a wavelet concentrating on the positive frequency axis for $\xi < 0$, is taken as $\hat{\psi}(\xi) = 0$, the Plancherel theorem, $W_s(a, b)$, which is a CWT according to s , is as in (6) we can rewrite it.

$$\begin{aligned} W_s(a, b) &= \frac{1}{2\pi} \int \hat{s}(\xi) a^{\frac{1}{2}} \hat{\psi}(a\xi) e^{ib\xi} d\xi \\ &= \frac{A}{4\pi} \int [\delta(\xi - \omega) + \delta(\xi + \omega)] a^{\frac{1}{2}} \hat{\psi}(a\xi) e^{ib\xi} d\xi \\ &= \frac{A}{4\pi} a^{\frac{1}{2}} \hat{\psi}(a\omega) e^{ib\omega} \end{aligned} \quad (6)$$

If $\hat{\psi}(\xi)$, concentrates around $\xi = \omega_0$, $W_s(a, b)$ will concentrate around $a = \frac{\omega_0}{\omega}$. However, the wavelet transform $W_s(a, b)$ will spread over a region around the horizontal line $a = \frac{\omega_0}{\omega}$ in the time scale plane. Although $W_s(a, b)$ spreads in a , the oscillation behavior in b represents the original frequency ω , regardless of the a value [16]. Estimation of instantaneous frequency $\omega_s(a, b)$ for each scale time pair (a, b) with $W_s(a, b) \neq 0$ in synchrosqueezing, is possible to reverse wavelet coefficients containing the same instantaneous frequency estimates. This is shown at (7).

$$\omega_s(a, b) = -i(W_s(a, b))^{-1} \frac{\partial}{\partial b} W_s(a, b) \quad (7)$$

Given the wavelet coefficients, SST $T_s(\omega, b)$ is located in (8). Where δ is a delta function and $A(b) = \{a; W_s(a, b) = 0\}$ and $\omega(a, b)$, are as defined in (7) above, for (a, b) to be $\in A(b)$ [15].

$$T_s(\omega, b) = \int W_s(a, b) a^{-\frac{3}{2}} \delta(\omega_s(a, b) - \omega) da \quad (8)$$

In the literature, it is seen that the WSST method is mainly used in signal processing studies in the biomedical field. It is seen that this method gives more successful results in terms of performance compared to some TF analysis methods available in the literature. An example of WSST of a sleep EEG signal is shown in Fig. 2.

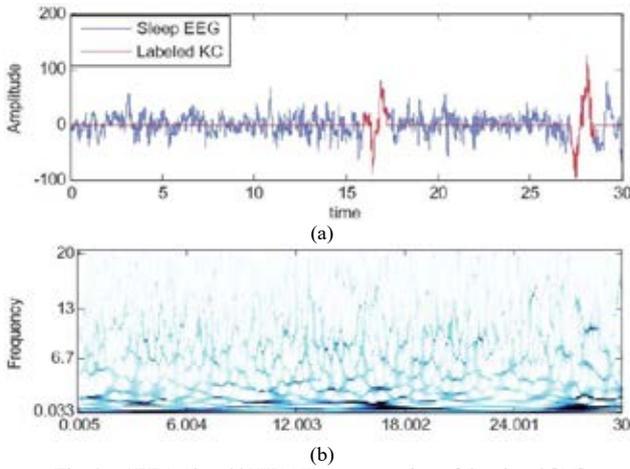


Fig. 2. a)EEG Signal b)WSST representation of the signal [17].

In the study conducted in 2013, a comparison of complete ensemble empirical mode decomposition (CEEMD) and WSST methods on seismic signals has been shown that the WSST method is attractive for high-resolution TF analysis of seismic signals. It was concluded that WSST and CEEMD perform equally well in seismic TFR, but the TFR obtained with WSST is based on more robust mathematics and is better than CEEMD in terms of speed performance [18]. Apart from a high-resolution TF analysis, this method affects classification success by using certain classification methods in studies for the diagnosis of diseases. In another study conducted by establishing a brain-computer interface for the detection of visual and hearing disorders in 2014, the WSST method was used to remove EEG parasites and to improve the classification process. WSST method was applied to the signals received from individuals and transferred to the P300 classification system. According to the results, WSST is more effective than the empirical mode decomposition (EMD) method, which is available in the literature and used in biomedical applications [19].

There are some applications of WSST for the detection of sleep disorders in the literature. In the study conducted in 2015, an effective algorithm was developed with 96.5% sensitivity and 98.1% specificity using WSST and a cumulative sum chart to detect the sleep spindle [20]. CWT is a TFR, the building block of WSST, but WSST is known to offer a more ideal TFR. In another application, WSST was used for the analysis and automatic detection of k-complex, a model that appeared in sleep EEG, and its comparison with CWT. In this comparison, it has been observed that WSST is superior to CWT in locating k-complexes' locations, time, and frequency properties [17]. In 2016, Rutkowski worked on the classification of sleep apnea. Due to the computational complexity of the EMD method, it was not appropriate to use this method in the online processing of EEG signals, considering the sensitive data-based and flexible band-pass filter features of the WSST method for processing nonlinear sleep signals. In addition to this method, it has used the Riemannian Geometry Failure Extraction method, a geometry-based classification method, and has developed an algorithm with 100% accuracy that successfully performs 6 sleep apnea classifications [21]. In a study in the following years, WSST was applied with a repeated TF estimation to provide a sound estimate of instantaneous breathing

frequency and to identify areas with/without sleep apnea/hypopnea events. Signal reconstruction was performed using inverse wavelet synchrosqueezed transform (IWSST) [22].

Recently, with the use of deep learning methods, it was used in addition to SST methods in the diagnosis and classification of diseases and effective success results were obtained. In 2020, Jarchi *et al.* used electromyogram (EMG) signals for entropy measurements in their study to diagnose disorders related to sleep disorders. The WSST method was applied to the ECG signals obtained and then IWSST was applied to reconstruct the signal. After obtaining ECG features such as instantaneous frequency and amplitude modulation, ECG signals were given to the deep neural network (DNN) architecture. The study provides a method with 72% accuracy for the classification of 4 different sleep problems [23]. An unbalanced data problem is one of the problems that may affect the success of the study. To prevent this, in a study to determine the sleep spindle in the same year, in addition to the WSST method, RUS-boost was used to prevent the problem, and the SST-RUS algorithm with a sensitivity of 76.9% was developed [24].

Some of the biomedical studies using WSST in the literature are on heart diseases. To detect potentially vital signs using ECG signals, in 2016, Zhao *et al.* conducted a study with effective results using this method [25]. Combining SST methods with different techniques can be used to improve SST methods. In a study in which WSST was used together with SVM to classify the cardiac disorder, high accuracy 99% and sensitivity 98% were obtained, and WSST was combined with a Wiener filter to provide the best noise suppressing performance [26]. The properties of the J wave, the decreasing curve of the QRS complex, are very important for the health of individuals. Any problem that may occur in this wave can cause sudden death. In 2018, Li *et al.* used the WSST method on ECG signals, taking into account this situation, so they observed the TF properties of the signals, the entropy and TF properties of the signals were given to Random Forest (RF), a classification method, and have developed a method with 96.9% accuracy [27]. In 2019, a method with effective classification results was developed by Ghosh and colleagues, using similar methods to detect 3 different heart conditions on polysomnography signals [28].

In a study where TFR of the FSST and WSST methods were transferred to a deep convolutional neural network (CNN), success rates with FSST and WSST were compared. FSST showed 100% success in both channels with 2D CNN, while WSST showed a lower success rate in channel 1 EEG [29].

C. Multivariate Synchrosqueezing Transform (MSST)

Recently, the concept of unidirectional modulated oscillation has been extended to the multivariable state using well-understood joint instantaneous frequency and bandwidth concepts to model the joint oscillation structure of a versatile signal [30]. In (9), a vector at each t is generated to give a multivariate analytical signal, $x_+(t)$.

$$x_+(t) = \begin{bmatrix} a_1(t)e^{i\phi_1 t} \\ a_2(t)e^{i\phi_2 t} \\ \vdots \\ a_N(t)e^{i\phi_N t} \end{bmatrix} \quad (9)$$

Here it represents the instantaneous amplitude and phase for each channel. The study in [31] suggested the common instantaneous frequency (power-weighted average of instantaneous frequencies of all channels) of multivariate data in (10).

$$\omega_x(t) = \frac{\Im\{x_+^H(t)\frac{d}{dt}x_+(t)\}}{\|x_+(t)\|^2} \quad (10)$$

Here, the symbol \Im indicates the imaginary part of a complex signal. While the joint/multivariate instantaneous frequency captures the combined oscillating dynamics of multivariate signals, the joint instantaneous bandwidth captures the deviations of the multivariate oscillations of the deviations from the instantaneous frequency in the joint and this is shown in (11).

$$v_x(t) = \frac{\left\| \frac{d}{dt}x_+(t) - i\omega_x(t)x_+(t) \right\|}{\|x_+(t)\|} \quad (11)$$

Adding (9) to (11) results in an expression for the square instantaneous bandwidth, which is shown in (12);

$$v_x^2(t) = \frac{\sum_{n=1}^N (a'_n(t))^2 + \sum_{n=1}^N a_n^2(t)(\phi'_n(t) - \omega_x(t))^2}{\sum_{n=1}^N a_n^2(t)} \quad (12)$$

An example of the MSST of EEG signals is shown in Fig. 3.

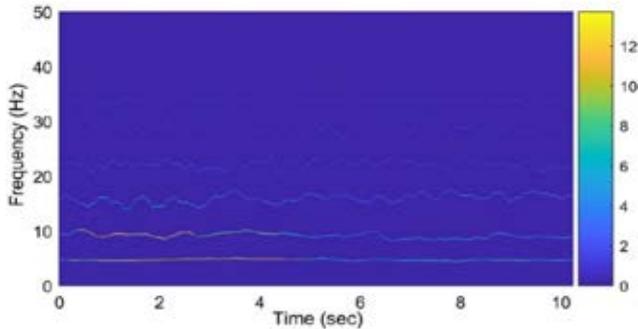


Fig. 3. MSST representation of the EEG signal [32].

MSST has multichannel signal processing and compact component localization capabilities, making it good for many areas. This method is used in the analysis of inter-channel dependencies that may arise from the data of multi-channel signals. As an example of MSST applications, the feasibility and performance of the multivariable WSST were compared with the univariate version and shown on EEG signals. Using linear SVM as a classifier resulted in the highest predictive accuracy rate of 93% among all emotional states in the MSST and univariate version [33]. Another study was analyzed using the MSST method to reveal oscillations from EEG records. Also, independent component analysis (ICA) and feature selection have been implemented to reduce high dimensional 2D TF distribution. The proposed MSST-ICA feature approach led to the conclusion that it gives 82.11% and 82.03% accurate rates for stimulation and valuation status recognition using an artificial neural network [32].

D. De-Shaped SST (dsSST)

dsSST provides a sharper TFR consisting of instantaneous frequency and amplitude modulation of the fundamental component using de-shape STFT [34]. Applying the dsSST

method to a channeled signal can adaptively isolate the high and low frequencies in the signal. In 2019, the dsSST method was used to extract important information from pneumographic signals. As a result of the study, it was observed that the application of the dsSST method was effective in extracting important information from the noisy signals [35].

III. DISCUSSION

The nonstationary feature of biosignals has been insufficient to analyze these signals of TF displays, and recently switched to the SST method, which offers the ability to analyze nonstationary signals. Unlike the TF display, SST offers a high-resolution TFR with sharp lines where unnecessary information is minimized for unstable signals. SST has a strong mathematical background compared to the HHT and EMD methods available in the literature. Thanks to this feature, it enables the original signal to be reconstructed after synchronization with a reverse synchronization process and one of the advantages it offers in filtering operations. It has been observed that the SST methods have been positively affecting the classification success of these techniques by using them together with machine learning and deep learning techniques. SST method, which is an improved version of TF notations, uses different mathematical methods such as FSST, WSST, MSST in biomedical signal processing studies in the literature. Table I shows the biomedical studies using different SST methods and their accuracy. Since there is not enough study on dsSST in the literature, dsSST was not mentioned in Table I, but it offers a sharper TFR.

TABLE I. COMPARISON TABLE OF SST IN BIOMEDICAL APPLICATIONS.

SST Types	APPLICATIONS			
	Method	Classification	ACC (%)	Ref.
FSST	5 fold 2D-CNN	Focal and Non-focal EEG	100	[29]
	CNN, Data Augmentation	Dysphonia voice	> 70	[36]
WSST	Riemannian geometry feature extraction, MDM	EEG, Sleep Apnea Detection	100	[21]
	SVM	ECG, Cardiac Disorder	98	[26]
	2D-CNN	Focal and Non-focal EEG	89.6(Avg)	[29]
MSST	RF classifier	PCG, Heart Valve Disorders	95.12	[28]
	RF classifier	ECG, J Wave Detection	94.1	[27]
	SVM	EEG, Emotional State	93	[33]
	ICA, t-test selection, ANN	EEG, Emotion Recognition	82.11	[32]
	SVD, AdaBoost Decision Tree	EEG, 2D and 3D Emotion	97	[37]

The SST method provides an ideal TFR. The analysis of nonstationary biomedical signals with TFR is inadequate, and SST methods are needed at this point. It is seen that different SST methods are used in biomedical signal processing applications in the literature. The difference in SST methods also affects the success rates of the study. Although the SST method provides high-resolution TF analysis for nonstationary signals, there are points where it is insufficient. It can be seen that using this method together with different methods contributes to success rates.

IV. CONCLUSION

In this paper, the different types of SST used in the literature are mentioned and a review of the biomedical applications of the SST methods has been made. It is seen that the detection and classification studies carried out together with SST show a high success. However, these studies mostly focused on the use of EEG and ECG signals as biosignals, and the WSST method was used more in the studies. More studies should be done in signal processing using different SST methods. The studies to be carried out by combining SST methods with different techniques will be promising to take into account the drawbacks and to overcome these drawbacks.

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