Trajectory Planning of Electronically Controlled Prosthesis by Using Third-Order Polynomial

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Abstract— **In this study, the mechanical model of the electronically controlled knee prosthesis is developed, forward and inverse kinematic analyses are performed, and trajectory tracking for joint space is obtained, respectively. The analyses were carried out considering a cycle of gait walking at a constant speed. Thus, position, velocity and acceleration data of the prosthesis of joint space required for the control of the prosthesis is obtained.**

Keywords —electronically controlled prosthesis, forward kinematics, inverse kinematics, trajectory planning, joint space, third-order polynomial.

I. INTRODUCTION

Nowadays people can lose their limbs because of various reasons. This deficiency can not only reduce the comfort of the person but also negatively affect psychologically. A large part of the negative effect is aesthetic visual anxiety. In some cases, the aesthetic image can also prevent the comfort of life. For this reason, prostheses have great prominence in human life. It is desirable that a good prosthesis has an indistinguishable image like a healthy person's limb and that it fulfills the desired function perfectly.

About 1.9 million people have lost their limbs on Earth due to various reasons. About 400,000 of these have been found to be amputations above the knee [1]. A lot of work is being done around the world to normalize their lives. Most of the prostheses sold in the market are mechanical prostheses with fixed joints and hinge joints. These prostheses help to maintain one's life, but are not as good as the comfort of a healthy person. For this reason, electronic controlled prostheses are of great importance for the aesthetic movement required by the person.

Electronically controlled prosthesis is a robotic system due to its offline operation and microcontroller control. It has three free joints. It is important to establish a proper trajectory in order to obtain the best performance in control of the system. Two different methods are used to create an offline trajectory tracking in literature: Cartesian and joint space. Because of necessity of completion of Heavy calculations in short time and errors that occur when taking inverse Jacobean, trajectory creation in Cartesian space is not preferred by robotic researchers [2,3]. By using inverse kinematics in joint space, joint position information is obtained. Trajectory generation in joint space is much easier than in Cartesian space because of considering only kinematic operations. Therefore, studies in joint space have been performed in this study.

II. METHOD AND MATERIAL

A. Motion Analysis in the Sagittal Plane

The movement of a leg for a cycle of walking consists of two phases; stance and swing. During these phases, knee and ankle angles changes constantly depending trajectory of the leg.

Figure.1 shows the sagittal plane gait phase angle changes in the knee and ankle.

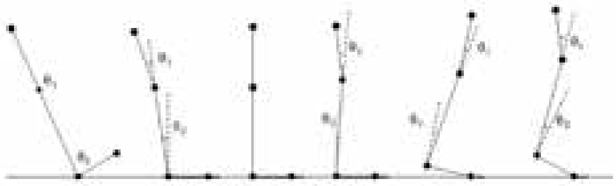

Figure.1 Stance phase of a gait action for a cycle on the flat surface in the sagittal plane

In the stance phase, the ankle joint is in the neutral position when the heel touches the ground. With the plantar flexion of the ankle joint, the foot base moves forward. The ankle joint moves 15 degrees with the contact of foot base to ground. When the foot base contact on the ground, it is middle of the stance phase. During separation of the heel from the floor, the ankle joint makes a movement of about 15 degrees of dorsal extension. During separation of the fingers from the ground, the angle between the tibia and the foot moves to other direction and the ankle enters the plantar flexion state of approximately 20 degrees [4].

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The swing phase begins with the fingers separating from the ground and ends with the heel touching the ground. As shown in Figure 2, this movement has three phases; acceleration, middle of swing phase and end of swing phase.

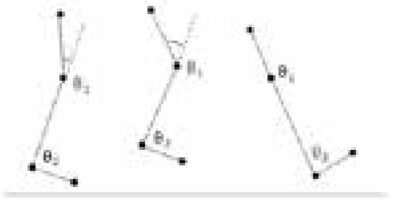

Figure.2 Swing phase of a gait action for a cycle on the flat surface in the sagittal plane

The walking cycle begins with the acceleration phase and the foot is removed from the floor. At this stage, hip and knee flexion increases and dorsiflexion occurs in the ankle. In the midst of the swing phase, the oscillated leg comes to the other leg. Hip and knee flexion increase, ankle moves dorsiflexion.

End of the swing phase starts when the released leg moves to in front of the other leg and goes on the foot touch the ground. At this time the knee is in extension and the ankle is in neutral position [4].

B. Kinematic Analysis Application

A mechanical model of the system has been created to perform kinematic analysis. The system exhibits a total of 3 free degree movements, one on the knee joint and two on the ankle.

In order to get the forward kinematics of the system, the coordinate systems as shown in Figure 3 were obtained and the D-H parameters of the model were determined using the Denavit-Hartenberg method.

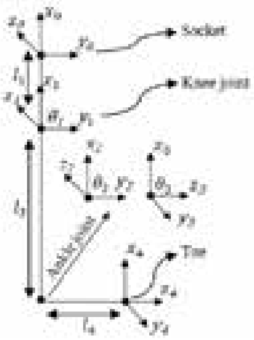

Figure.3 Joint coordinate systems of the prosthesis

Using the D-H parameters, the general transformation matrix is calculated.

$$ {}^0_4T = {}^0_1T\,{}^1_2T\,{}^2_3T\,{}^3_4T \tag{1}$$

θ_1 and θ_2 equations are obtained by computing the inverse kinematics of the system from Equations (2) and Equations (3).

$$\theta_1 = \mathrm{Atan2}(\mathrm{p_y}, \mathrm{p_x}) \pm \mathrm{Atan2}\left(\sqrt{\mathrm{p_y}^2 + \mathrm{p_x}^2 - (\mathrm{l_2} + \mathrm{l_4 s\Theta_2})^2}, \mathrm{l_2} + \mathrm{l_4 s\Theta_2}\right) \tag{2}$$

$$\theta_2 = \mathrm{Atan2}(\mathrm{p_y}, \mathrm{p_x}) \pm \mathrm{Atan2}\left(\frac{p_x{}^2+p_y{}^2-l_2{}^2-l_4{}^2}{2l_2l_4}, \pm\sqrt{1-\left(\frac{p_x{}^2+p_y{}^2-l_2{}^2-l_4{}^2}{2l_2l_4}\right)^2}\right) \tag{3}$$

Notice that the angle θ_3 of the system is not obtained. Because of this reason is that the angle θ_3 makes the movement of the ankle eversion and inversion. This movement of the joint has no effect on the position in the Cartesian space of the leg and the getting of the trajectory.

C. Third-Order Polynomial

Third order or higher degrees of polynomials are used for trajectory planning in joint space. In order to move the end-effector of a robot from the starting point to the desired point, the inverse kinematics is calculated firstly. Thus, the position and orientation of the starting and destination points are obtained in terms of the joint angles. In this project, end-effector is toe and third order polynomial is used for trajectory planning [5].

The initial position of the end function at t is $\theta(0) = \theta_1$ and the target position at t_f is $\theta(t_f) = \theta_f$.Initial and final velocities $\dot{\theta}(0) = 0$ and $\dot{\theta}(t_f) = 0$ [5].

Midpoints in the joint space of the hip, knee, and ankle were obtained by inverse kinematics in a gait cycle and are presented in Table 1.

In the equations, ct and st mean $cost$ and $sint$ respectively. Equations (4) and (7) give the position expression depending on the angles θ_1 and θ_2 of the joint, respectively. Equations (5) and (8) are obtained by taking the first derivative of these equations respectively and give this speed information. By taking the second derivative, Equation (6) and Equation (9), which are acceleration equations, are obtained.

$$\theta_1(t) = s_0 + s_1t + s_2t^2 + s_3t^3 \tag{4}$$

$$\dot{\theta}_1(t) = s_1 + 2s_2t + 3s_3t^2 \tag{5}$$

$$\ddot{\theta}_1(t) = 2s_2t + 6s_3t \tag{6}$$

$$\theta_2(t) = s_0 + s_1t + s_2t^2 + s_3t^3 \tag{7}$$

$$\dot{\theta}_2(t) = s_1 + 2s_2t + 3s_3t^2 \tag{8}$$

$$\ddot{\theta}_2(t) = 2s_2t + 6s_3t \tag{9}$$

III. CASE STUDY

In this study, the trajectory of the hip, knee and ankle joints was extracted in a walking cycle. A gait cycle has been accepted as 1 second. The trajectory consists of a total of 20 midpoints and therefore 19 segments. The correspondences of these midpoints in joint space are given in Table 1.

Midpoints	Hip (degree)	Knee (degree)	Ankle (degree)
1	18	13	-2
2	17,5	23	-8
3	15,5	26	-5
4	12	25	1
5	8	23	3
6	4	20,5	5
7	1,5	19	7
8	-2	19	9
9	-4,5	20	11
10	-7	24	9
11	-8	31	0
12	-5	43	-15
13	1	58	-24
14	9	65	-21
15	14	66	-14
16	17	59	-8
17	20	48	-5
18	20	36	-3
19	18	25	-1
20	16	8	-1

Table.1 Joint angles for trajectory tracking in the joints space

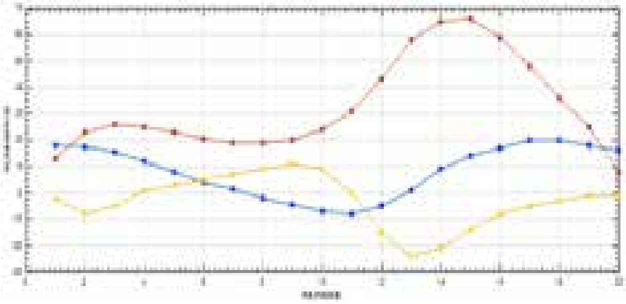

Figure.4 Position profile of hip, knee and ankle joints for trajectory tracking in the joints space (Each midpoint is scaled to 50 ms.)

Mid points	Hip angle	Position (degree)	Velocity (degree/second)	Acceleration (degree/second^2)
1	18	17,98309728	-0,6757632	-13,494528
2	17,5	17,46337224	-1,4654656	-29,330624
3	15,5	15,50290856	0,1163136	2,324544
4	12	12,00740856	0,2963136	5,924544
5	8	8,01096952	0,4387712	8,774848
6	4	4,012	0,48	9,6
7	1,5	1,51059144	0,4236864	8,475456
8	-2	-1,99156096	0,3375424	6,749696
9	-4,5	-4,49043904	0,3824576	7,650304
10	-7	-6,9925	0,3	6
11	-8	-7,99390856	0,2436864	4,875456
12	-5	-5,00075616	-0,0301696	-0,598784
13	1	0,98818288	-0,4726272	-9,449088
14	9	8,98012192	-0,7950848	-15,899392
15	14	13,97881712	-0,8473728	-16,950912
16	17	16,98687808	-0,5249152	-10,500608
17	20	19,991	-0,36	-7,2
18	20	19,99381712	-0,2473728	-4,950912
19	18	18,00187808	0,0750848	1,499392
20	16	16,006	0,24	4,8

Table.2 Position, velocity and accelaration of hip joints for trajectory tracking in the joints space

Mid points	Knee angle	Position (degree)	Velocity (degree/second)	Acceleration (degree/second^2)
1	13	12,98309728	-0,6757632	-13,494528
2	23	22,96337224	-1,4654656	-29,330624
3	26	26,00290856	0,1163136	2,324544
4	25	25,00740856	0,2963136	5,924544
5	23	23,01096952	0,4387712	8,774848
6	20,5	20,512	0,48	9,6
7	19	19,01059144	0,4236864	8,475456
8	19	19,00843904	0,3375424	6,749696
9	20	20,00956096	0,3824576	7,650304
10	24	24,0075	0,3	6
11	31	31,00609144	0,2436864	4,875456
12	43	42,99924384	-0,0301696	-0,598784
13	58	57,98818288	-0,4726272	-9,449088
14	65	64,98012192	-0,7950848	-15,899392
15	66	65,97881712	-0,8473728	-16,950912
16	59	58,98687808	-0,5249152	-10,500608
17	48	47,991	-0,36	-7,2
18	36	35,99381712	-0,2473728	-4,950912
19	25	25,00187808	0,0750848	1,499392
20	8	8,006	0,24	4,8

Table.3 Position, velocity and accelaration of knee joints for trajectory tracking in the joints space

Mid points	Ankle angle	Position (degree)	Velocity (degree/second)	Acceleration (degree/second^2)
1	-2	-1,99998272	-0,6757632	-13,494528
2	-8	-8,00001776	-1,4654656	-29,330624
3	-5	-5,00000144	0,1163136	2,324544
4	1	0,99999856	0,2963136	5,924544
5	3	2,99999952	0,4387712	8,774848
6	5	5	0,48	9,6
7	7	7,00000144	0,4236864	8,475456
8	9	8,99999904	0,3375424	6,749696
9	11	11,00000096	0,3824576	7,650304
10	9	9	0,3	6
11	0	0,00000144	0,2436864	4,875456
12	-15	-14,9999962	-0,0301696	-0,598784
13	-24	-23,9999971	-0,4726272	-9,449088
14	-21	-20,9999981	-0,7950848	-15,899392
15	-14	-14,0000029	-0,8473728	-16,950912
16	-8	-8,00000192	-0,5249152	-10,500608
17	-5	-5	-0,36	-7,2
18	-3	-3,00000288	-0,2473728	-4,950912
19	-1	-1,00000192	0,0750848	1,499392
20	-1	-1	0,24	4,8

Table.4 Position, velocity and accelaration of ankle joints for trajectory tracking in the joints space

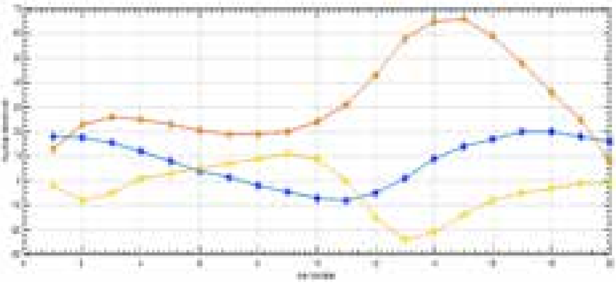

Figure.5 Position profile of hip, knee and ankle joints which is obtained by using third-order polynomial for trajectory tracking in the joints space (Each midpoint is scaled to 50 ms.)

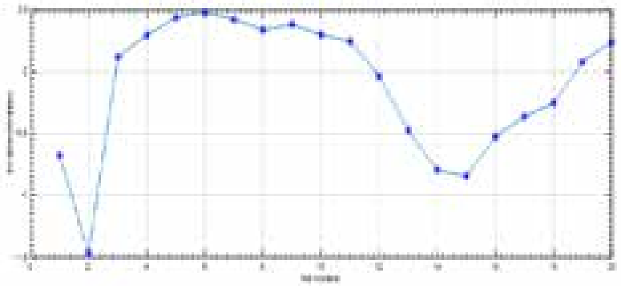

Figure.6 Velocity profile of hip, knee and ankle joints which is obtained by using third-order polynomial for trajectory tracking in the joints space (Each midpoint is scaled to 50 ms.)

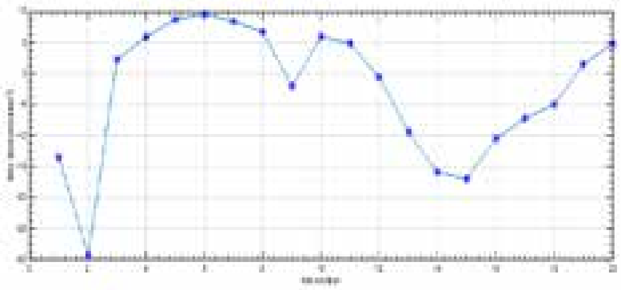

Figure.7 Acceleration profile of hip, knee and ankle joints which is obtained by using third-order polynomial for trajectory tracking in the joints space (Each midpoint is scaled to 50 ms.)

IV. CONCLUSIONS

In this project, the mechanical model of the prosthetic system was created firstly. Forward and inverse kinematic analysis was performed by determining the D-H parameters of the system. Thus, joint angles required for the position and orientation of the prosthesis in Cartesian space was obtained. The movement on the Cartesian space was carried out for a cycle of walking. It was assumed that a cycle walking action consisting of swing and stance phase was completed in 1 second. This motion was divided into 20 midpoints and the trajectory of the prosthesis was obtained. Position, velocity and acceleration are presented in tables and graphs using third-order polynomial. This data will be used for trajectory optimization in the control of the system so that an electronically controlled prosthesis to be produced can show the closest motion to a normal human motion.

ACKNOWLEDGEMENT

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