Medikal Amaçlar İçin Önerilen Yeni Bir Hibrit Robotun Ters Kinematik Çözümü

Inverse Kinematics Solution of a New Hybrid Robot Manipulator Proposed for Medical Purposes

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Anahtarkelimeler—Medical robots, serial manipulators, parallel manipulators, hybrid manipulators, inverse kinematics.

Abstract—In this paper, a new hybrid robot manipulator is proposed for medical purposes. This hybrid robot manipulator has been formed by combination of a serial and a parallel robot manipulator. The inboard joints of new hybrid robot consist of 3-DOF SCARA type robot manipulator that performs positioning of the end effector. The outboard joints of the new hybrid robot consist of Stewart platform parallel manipulator which is used for orientation of the end effector only. Afterwards, the inverse kinematics of this new hybrid robot manipulator is obtained by using DH method. A numerical example is also given in order to demonstrate the working of the model.

Keywords—Medical robots; serial manipulators; parallel manipulators; hybrid manipulators; inverse kinematics.

I. INTRODUCTION

Medical robots have increasingly interested the medical society for the last decades since they have been used for several clinical applications such as neurosurgery, orthopedics, urology, radiosurgery, and cardiac surgery[1]. Medical robots provide some important advantages over traditional methods such as smaller incision, accuracy, more stabilization compared to human hand, shorter operation and recovery time[2][3][4][5]. Medical robotic systems can be classified into three generations. The first generation medical robots were modified from industrial robots the eighties. They were mostly used for orthopedics, neurosurgery radiology and radiotherapy. The medical robot used in orthopedics especially cut and drill the bones for implants while the medical robot used in neurosurgery accurately position surgical instruments inside the patient head[6]. The second generation medical robots were developed with special designs in order to perform surgical operation the nineties. They were used especially for orthopedics, neurosurgery, radiology, and minimally invasive surgery. The medical robot used in minimally invasive surgery holds a camera arm or mechanical arms having necessary surgical instruments. The surgeon uses these mechanical arms to perform the operation.
The third generation medical robots were developed in order to perform surgical and medical operations which are extremely difficult for medical experts in present times. They are used especially for orthopedics, interventional radiology and cardio-vascular issues. Table 1 gives the historical development of robot manipulators (serial or parallel) for medicine [7]. Serial and parallel robot manipulators in Table 1 perform both position and orientation at the same time. The serial robots in Table 1 have Euler wrists in general. This paper presents a new hybrid robot manipulator that takes advantages of both serial and parallel manipulators to get better medical results. Table 2 presents a comparison between the characteristics of the serial and parallel robot manipulators [8]. As can be seen in Table 2, serial robot manipulators have wider workspace compared to parallel manipulators. Therefore, inboard joints of new hybrid robot manipulator comprise 3-DOF SCARA type robot manipulator that performs positioning of the end effector only. Parallel robot manipulators have higher kinematic features (stiffness, accuracy, velocity acceleration) and better dynamic features (smaller inertia) than serial manipulators. Therefore outboard joints of the new hybrid robot manipulator consist of Stewart platform parallel manipulator which is used for orientation of the end effector only. Inverse kinematics is one of the major issues of robotics and extensively studied in the literature [9][10][11][12]. Erturun designed a hybrid robot including combination of serial and parallel robots [13][14]. The positioning part of the hybrid robot includes four revolute joints. The third and fourth axes of this positioning part especially were added to the hybrid robot in order to provide hexapod more flexible positioning. In this new case, this hybrid robot includes at least ten actuators to perform a task. Fourth joint is a redundant joint that makes kinematic, dynamics and control issues more complex. However the hybrid robot proposed in this study has a simpler mechanism having less than ten actuators. I addition, Erturun used Cosmos Motion software to solve inverse kinematics numerically. However, in this study the inverse kinematics of the new hybrid robot is solved analytically.

### Tables

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<tr>
<td>Accuracy</td>
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<td>High</td>
</tr>
<tr>
<td>Workspace/Robot Size Ratio</td>
<td>High</td>
<td>Low</td>
</tr>
</tbody>
</table>

Table 2. Features of serial and parallel robot manipulators

### B. Inverse Kinematics of New hybrid Robot

The inverse kinematics of this new hybrid robot manipulator can be obtained by using DH method [12]. In this method, coordinate systems are attached to each joint as in figure 2. The equations between 5 and 12 are used in [10] are used for solving the inverse kinematics of new hybrid robot.

Since Stewart platform parallel manipulator is used for orientation of the end effector, it provides a fix distance along z-axis only. The distances along x and y axes are adjusted to zero. In this case, the hybrid manipulator is forced to perform motion like as 6-DOF serial manipulator having first three joints forpositioning and last three joints for orientating. The fix distance along z-axes can be optimized in such a way that the 6 active actuator of Stewart platform provides maximal orientation about x, y and z-axes. In this paper an approximate
fix distance is accepted. Since the new hybrid robot manipulator comprises two substructures in terms of a 3-DOF serial robot manipulator unit and 6-DOF Stewart platform manipulator unit, the inverse kinematics is solved in two stages. Stage 1 includes in the solution of 3-DOF serial robot manipulator and second stage consists of the solution of 6-DOF Stewart platform manipulator. The radiuses of the base and moving platforms are denoted as \( r_b \) and \( r_m \), respectively. Separation angles between each leg of the Stewart platform are 60 degrees. Rotation Roll-Pitch-Yaw angles set is performed in terms of fixed reference coordinate system (base frame). The forward kinematics matrices of SCARA robot

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(1)

The forward kinematics matrices of end-effector attached to the moving platform of Stewart platform manipulator.

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(5)

Where \( w_z \) is the fix distance along z-axis, and \( s_x, s_y, s_z \) are the position vector of SCARA robot manipulator.

\[
s_x = l_x (\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2) + l_1 \cos \theta_1
\]

(11)

\[
s_y = l_y (\cos \theta_1 \sin \theta_2 + \cos \theta_2 \sin \theta_1) + l_1 \sin \theta_1
\]

(12)

\[
s_z = h + d_z
\]

(13)

The inverse kinematics solution of SCARA robot is obtained as follows.

Taking square of (1,4) and (2,4) matrix elements of each sides in eq.6 and adding the results, after simplification the revolute joint variable \( \theta_2 \), can be computed as.

\[
\theta_2 = A \tan \left( \frac{\sqrt{1 - k^2}}{k} \right)
\]

(14)

\[
\theta = A \tan \left( -p_y - p_x \right) + \frac{\pi}{2} - \frac{1}{2} \sin \theta_2
\]

(15)

Prismatic joint variables \( d_1 \) can be attained by equating (3,4) matrix elements of each sides in equation 6.

\[
d_1 = p_z - h
\]

(16)

The inverse kinematics solution of Stewart platform is given as

\[
d_i = (p_i - b_i) + (p_i - b_i) + p_i^2 \quad i = 1,2,\ldots,6.
\]

(17)

where

\[
b_i = r_i \cos(60^\circ)
\]

(18)

\[
b_{i} = r_i \sin(60^\circ)
\]

(19)

\[
p_i = m_r \cos \beta \cos \gamma - m_r \cos \beta \sin \gamma
\]

(20)

\[
p_{i} = m_r (\cos \alpha \sin \gamma \sin \alpha \sin \beta \cos \gamma)
\]

(21)

\[
m_{i} = m_r (\cos \alpha \sin \gamma \sin \alpha \sin \beta \sin \gamma)
\]

(22)

\[
p_{i} = w_z + m_r (\sin \alpha \sin \gamma - \cos \alpha \sin \beta \cos \gamma)
\]

(23)

\[
m_{i} = w_z \sin(60^\circ)
\]

(24)

\[
m_{i} = w_z \cos(60^\circ)
\]

(25)

The flowchart of the inverse kinematics for new hybrid robot manipulator can be summarized as follows:

1. Given the position vector \( (P_{HYBRID}) \) of end-effector in terms of base frame of hybrid robot manipulator.
2. Given orientation (Roll-Pitch-Yaw angle set) of end-effector in terms of base frame of Stewart platform.
3. Find the inverse kinematics of Stewart platform manipulator \( (d_1, d_2, d_3, d_4, d_5, d_6) \).
4. Find position vector \( P_{STWR} \) of end-effector by using \( \mathbb{T}^{stw} \) transformation matrix in terms of base frame of Stewart platform where \( s_x, s_y, s_z \) are equal to zero.
5. Subtract position vector \( P_{STWR} \) from the position vector \( P_{HYBRT} \) in order to find the position vector of the SCARA manipulator \( (s_x, s_y, s_z) \).
6. Find the inverse kinematics of SCARA robot manipulator \( (\theta_1, \theta_2, d_i) \).

C. Examples

In this section, 3 numerical examples are provided in order to illustrate the methodology used for the inverse kinematics solution of new hybrid robot manipulator. The link lengths of SCARA robot manipulator \( l_1=50cm \) and \( h_i=0 \). The radiuses of the base and moving platforms are chosen as \( r_b=20cm \) and \( r_s=10cm \) respectively. Link length of the end-effector is chosen as \( e_{ef}=10cm \). The fix distance \( w_s \) is chosen as 20 cm.

1) Example

The position vector of end-effector in terms of base frame of hybrid robot is chosen as \( P_{STWR} = [93.3013 \ 20 \ 43.6603]^T \).

The Roll-Pitch-Yaw orientation angles of the end-effector in terms of base frame of Stewart platform manipulator is selected as \( R_{xyz} = R_x(\alpha)R_y(\beta)R_z(\gamma) = (30, 0, 0) \). In this case, the inverse kinematics of Stewart platform manipulator is obtained as \( d_1 = 22.3607, d_2 = 26.7095, d_3 = 26.7095, d_4 = 22.3607, d_5 = 19.1569, d_6 = 19.1569 \). The position vector of the end-effector in terms of base frame of Stewart platform is obtained as \( P_{STWR} = [0 \ -5 \ 28.6603]^T \). The position vector of the SCARA manipulator is computed as \( SCARA(s_x, s_y, s_z) = P_{STWR} - P_{HYBRT} = [93.3013 \ 25 \ 15]^T \). The inverse kinematics of SCARA robot manipulator is obtained as \( \theta_1 = 0, \theta_2 = 30, d_i = 15 \). The schematic diagram of example 2 is given in Figure 5.

III. CONCLUSIONS

A new hybrid robot manipulator is proposed for medical purposes in this study. Superior features of serial SCARA robot manipulator and parallel Stewart platform robot manipulators are used for designing new hybrid robot manipulator. First three joints of SCARA robot manipulator is used for positioning while the Stewart platform robot manipulators are used for orientating the new hybrid robot manipulator. The inverse kinematics of this new hybrid robot manipulator is solved. Three successful numerical examples are provided to illustrate usefulness of the proposed the new hybrid robot manipulator. These numerical examples illustrates that this new hybrid structure can be designed and manufactured.

Figure 5. Schematic diagram of example 2

REFERENCES